DETECTION METHOD FOR WIDEBAND SHORT TIME CHIRP SIGNALS

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ABSTRACT

Chirp signals arise in many applications of signal processing. In this paper, we address the problem of detection of chirp signals that are encountered in a bistatic radar which we are developing for remote sensing of cosmic ray induced air showers. The received echoes from the air showers are wideband and characterized by very short sweep periods. This makes our astrophysical problem a challenging one, since a very short sweep period is equivalent to a very low energy chirp signal. We propose a rake-like receiver which consists of a set of filters matched to different chirp rates within a range of interest. We examine the detection capability of the proposed structure through extensive numerical analysis. We also report an implementation of the proposed receiver on a National Instruments FlexRIO FPGA board and evaluate its performance in a laboratory setting.

***Index Terms*—** Radar, Bistatic radar, chirp signal detection

I. INTRODUCTION

I-A. Motivation

Earth is being bombarded by energetic cosmic radiation, likely created in the Universe's most violent processes. Current understanding of cosmic rays comes from detectors covering thousands of square kilometers of the Earth's surface and costing tens of millions of dollars, thus the sheer scale of these observatories is becoming a limiting factor to our understanding and a major hindrance to pursue the research in high energy cosmic rays. Our research group has been working on a novel cosmic ray observatory based on such a new technique: the remote sensing via bistatic radar technology of cosmic ray induced extensive air showers [1]. This technique is promising as if successful, it will allow the next generation of cosmic ray observatories to be built at a fraction of the cost required by current technologies. According to our research, chirp signals are expected to be received from air shower radar echoes. Unlike existing chirp applications, our echoes are expected to be short-time frequency sweeps (order of microseconds) and characterized by undetermined high frequency rates within an approximated wideband.

I-B. State-of-the-art

Chirp signals are ubiquitous in nature. They can be observed in many areas, such as geophysics [2], underwater explorations [3], and gravitational waves in astrophysics [4]. Also, they are highly used in different areas of signal processing, such as sonar [5], radar [6], and spread spectrum communications [7], [8]. Some of these applications rely on chirp signal transmission as in the case of sonar [5], while others model the received signal after doppler spread as chirp signals, e.g., in synthetic aperture radars (SARs) [9], and heart sound signals [10]. In the literature, various techniques have been developed for the estimation of chirp parameters including the doppler frequency shift [11] and doppler frequency rate [12]. In the current phase of our application, our main interest lies in the detection of the received chirp echoes produced by cosmic ray induced air showers. Parameter estimation methods is part of our planned research and thus will be reported in our future publications.

Our problem is characterized by several aspects that make it unique and different from all published papers related to chirp signals. We are interested in the detection of chirp echoes of undetermined frequency rates within a relatively wide band. Moreover, the expected chirp events occur infrequently and are very short in duration, 0.2 to 10 μs each. In addition, our equipment should operate within signal-to-noise ratio (SNR) values in the range of a few dB or, some times in a negative dB range. In this paper, we propose a rake-like receiver that consists of a bank of matched filters. The numerical simulations and laboratory tests, that we perform, shows that the proposed receiver works very well, even when SNR is negative. We also use a National Instruments FlexRIO FPGA module as a high-performance custom hardware to implement the proposed receiver unit. Experimental results also demonstrate the high performance in a practical setting.

This paper is organized as follows. In Section II, we explain our system model. In Section III, we evaluate the performance of our proposed receiver through a numerical analysis study. In Section IV, we briefly discuss the system implementation and demonstrate a test that shows system performance. Finally, conclusions are given in Section V.

II. SYSTEM ARCHITECTURE

We assume that the signal of interest is a down-chirp signal that has a duration T_c seconds with a constant amplitude, a start (high) frequency f_{H}, center frequency f_{C}, end (low) frequency f_{L}, and chirp rate κ MHz/sec. Such a chirp signal is mathematically written as

$$c(t) = \mathrm{rect}\left(\frac{t}{T_c}\right)\cos(2\pi f_{\mathrm{C}} t - \pi\kappa t^2). \tag{1}$$

where $\mathrm{rect}(x)$ is the rectangular function and t is the time variable in seconds. We assume that f_{H} and f_{L} are known constants, however, the chirp rate parameter κ is unknown. The receiver input is assumed as

$$x(t) = c(t - T_o) + \nu(t) \tag{2}$$

where T_o is the delay due to channel and $\nu(t)$ is white Gaussian noise with variance of σ_ν^2. We wish to identify the presence of $c(t)$ in a finite duration of $x(t)$ in the interval $(0, T)$. Moreover, we assume that when $c(t - T_o)$ is present in this interval, the interval $(T_o - \frac{T_c}{2}, T_o + \frac{T_c}{2})$ is a subinterval of $(0, T)$.

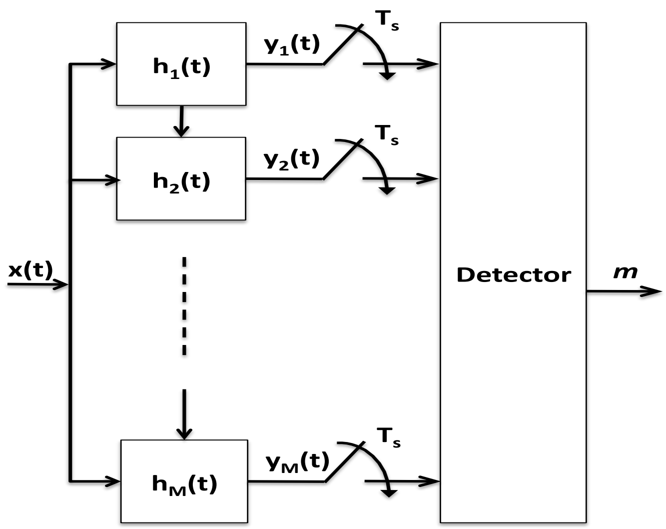

Fig. 1. Rake-like receiver structure based on matched filter bank

The scope of this paper is limited to the presence of the chirp signal $c(t)$, without any attention to value of the chirp rate κ. To this end, we propose to use a rake-like receiver consisting of a bank of filters matched to a number of quantized chirp rates, κ_1, κ_2, $\cdots$, κ_M. Fig.1 presents a block diagram of such a receiver.

We can consider the impulse response of the m^{th} matched filter in the receiver as [13]

$$h_m(t) = \begin{cases} c_m^*(-t), & 0 \le t \le T_m \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where $c_m(t)$ is the corresponding chirp waveform of the m^{th} matched filter with chirp rate κ_m and duration T_m. Accordingly, the output of the m^{th} matched filter is defined as

$$y_m(t) = \int_{-\infty}^{\infty} x(\tau) h_m(t-\tau) d\tau \ . \tag{4}$$

For an easier analysis of the output signal component, we derive the output in the equivalent baseband domain. Hence, we need to define the equivalent low-pass chirp signal as

$$c_{T_c}(t) = \mathrm{rect}\left(\frac{t}{T_c}\right) e^{-j\pi k t^2}. \tag{5}$$

We define the output signal component of the m^{th} MF in the equivalent low-pass domain as

$$\hat{y}_m^{'}(t) = \frac{1}{2}\int_{-T_c}^{T_c} c_{T_c}(\tau) c_{T_c}^*(-\tau) d\tau. \tag{6}$$

This integral can be evaluated as

$$\hat{y}_m^{'}(t) = \begin{cases} \frac{e^{j\frac{\pi\kappa\kappa_m t^2}{\kappa-\kappa_m}}}{\sqrt{8(\kappa-\kappa_m)}}\Big(\mathrm{F}^*(\theta_1(t)) + \mathrm{F}^*(\theta_2(t))\Big), & -T_c \le t \le T_c \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where

$$\theta_1(t) = \sqrt{\frac{\kappa-\kappa_m}{2}}\left(T_m - \frac{2\kappa}{\kappa-\kappa_m}|t|\right),$$

$$\theta_2(t) = \sqrt{\frac{\kappa-\kappa_m}{2}}\left(T_m + \frac{2\kappa_m}{\kappa-\kappa_m}|t|\right)$$

and $\mathrm{F}^*(x)$ is Fresnel conjugate complex integral that can be defined in terms of Fresnel cosine and sine integrals as [[14], pp.887, Eq. (8.250)]

$$\mathrm{F}^*(x) = \mathrm{C}(x) - j\,\mathrm{S}(x) \tag{8}$$

where

$$\mathrm{C}(x) = \frac{1}{\sqrt{2\pi}}\int_0^x \cos(t^2) dt \tag{9}$$

$$\mathrm{S}(x) = \frac{1}{\sqrt{2\pi}}\int_0^x \sin(t^2) dt \ . \tag{10}$$

Finally, the chirp related part of $y_m(t)$ is

$$\hat{y}_m(t) = \mathrm{Re}\{\hat{y}_m^{'}(t) e^{j2\pi f_C t}\}. \tag{11}$$

III. NUMERICAL ANALYSIS

In this section, we study the detection performance of our proposed receiver. We aim to optimize the choice of system parameters for maximizing the ability of the receiver to discern between signal and noise. Due to complexity of expression in (11) and, thus, any subsequent derivations, we resort to a numerical analysis to develop an in-depth understanding of the proposed chirp detector.

We let $T = 1$ second. We use σ_m^2 to denote the noise variance at the output of a filter matched to the chirp rate κ_m. We also use σ_m as the base unit to express the signal level at the output of the matched filters. The signal-to-noise ratio (SNR) at the input to the matched filter bank is defined as the ratio of the chirp signal power over the noise power, viz.,

$$\mathrm{SNR_{in}} = \frac{a^2}{2\sigma_\nu^2}. \tag{12}$$

According to our system environment, we expect low SNR level at the receiver input as low as -10 dB. We assume that the chirp rate κ is a uniformly distributed random variable in the range of 1 MHz/μs to 11 MHz/μs.

We consider a set of five matched filters with chirp rate parameter κ_m logarithmically distributed within the interval [1 MHz/μs, 11 MHz/μs] as depicted in Fig. 2. In Fig. 3, we graphically demonstrate the output of matched filter with an arbitrary κ_m that lie in the mentioned interval. The output of the filter is depicted for a range of input signals with chirp rates, logarithmically distributed over the interval $[\kappa_{m-1}, \kappa_{m+1}]$, for the case where $\mathrm{SNR_{in}} = 0$ dB. This result clearly shows that a matched filter with parameter κ_m correlates well with chirp signal whose chirp-rate parameter κ is within a range from κ_m. Specifically, chirp signal with $\frac{\kappa_m}{\sqrt{\Delta}} < \kappa < \kappa_m\sqrt{\Delta}$ results in samples with an absolute value greater than $6\sigma_m$. Even those values of κ that are out of the above range still produce reasonably large peaks.

We notice that the five matched filters results in the same behavior for input chirp signals with $\frac{\kappa_m}{\sqrt{\Delta}} < \kappa < \kappa_m\sqrt{\Delta}$. For numerical illustration of the level of signal peaks resulting from chirp signals at the outputs of a matched filter bank, Table I presents

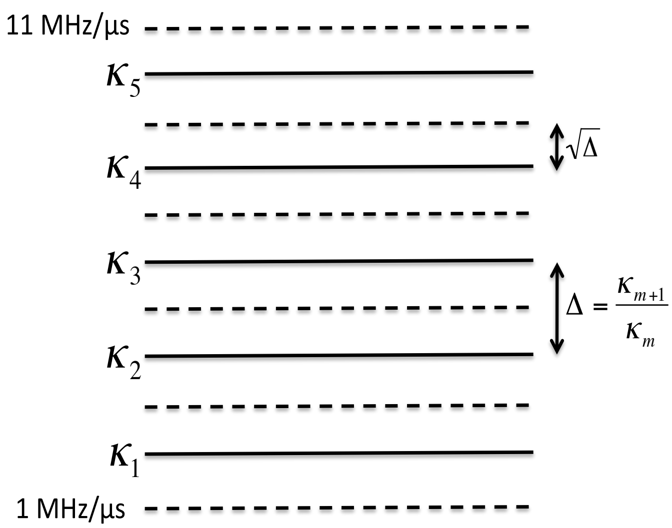

Fig. 2. Chirp rate parameter logarithmic distribution over the set of matched filters

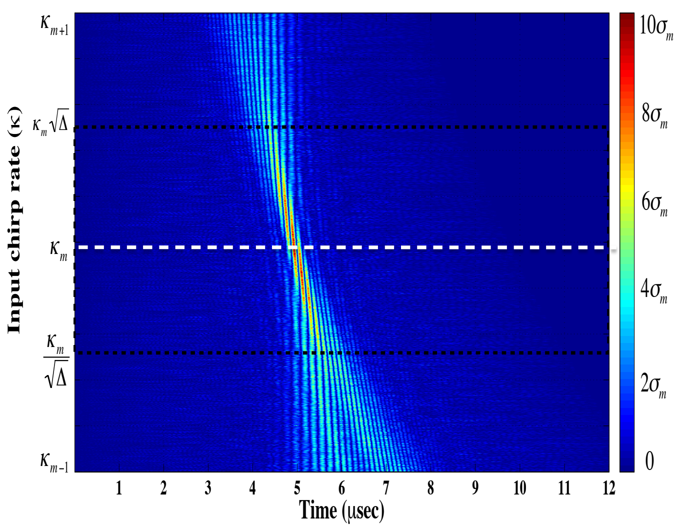

Fig. 3. Color map indicating the detection of a range of chirp rates using a single matched filter

some signal statistics. We assume that for each case the incoming chirp signal has the same rate as the respective matched filter, i.e., $\kappa = \kappa_m$. For each matched filter, we define a set of threshold value γ_m as indicated in Table I. We count the number of output samples of each matched filter, arising from the chirp signal, that pass the threshold level γ_m. The presented results are for the case where $\text{SNR}_{\text{in}} = 0$ dB. We can deduce that matched filters with parameter κ_m, logarithmically distributed within the interval [1 MHz/μs, 11 MHz/μs], results in approximately the same number of samples that exceeds reasonable threshold values in terms of σ_m. This result indicates that logarithmic distribution of κ_m can be considered as a near-optimal way for uniformly distributed κ. Moreover, results shown in Table I clearly show that even in a very

Table I. Signal statistics at the outputs of a bank of matched filters at SNR = 0 dB

γ_m	κ_m (MHz/μs)				
	κ_1	κ_2	κ_3	κ_4	κ_5
$12\sigma_m$	13	11	12	13	12
$10\sigma_m$	18	19	18	18	20
$8\sigma_m$	22	24	22	22	26
$6\sigma_m$	26	26	28	26	29
$4\sigma_m$	30	39	45	31	48

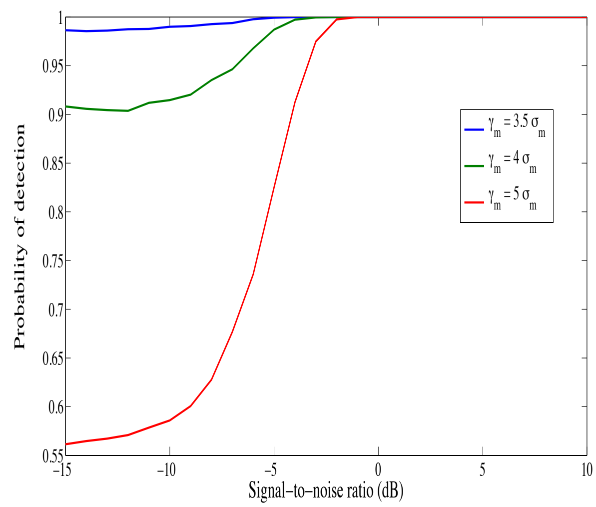

Fig. 4. Probability of detection versus signal-to-noise ratio over a range of threshold values

noisy environment, the proposed rake-like receiver allows one to choose a value of γ_m that can detect the presence of a chirp, while providing a very low probability of false alarm, i.e., reports the presence of a chirp when there is no chirp in the received signal.

Considering uniformly distributed κ in the range of [1 MHz/μs, 11 MHz/μs], we numerically calculate probability of detection over a range values of γ_m. In our expected SNR regime (-10 dB $\leq$SNR $\leq$ 5dB), we can see 100 percent detection efficiency for threshold values less than or equal $3\sigma_m$. Fig. 4 shows probability of detection for higher threshold values. Clearly at low SNR values, we can conclude that the higher the threshold is set, the detection efficiency decreases.

IV. SYSTEM IMPLEMENTATION

Fig. 5 shows the basic elements of the receiver structure. We utilize the NI-5761 adapter module with 500 MHz signal bandwidth and a sample rate up to 250 Million samples per second. Our system-on-chip design is implemented over the high performance Virtex-5 FPGA which is integrated with PXIe interface for host connectivity. In the demonstration, we use Agilent ESG vector signal generator with a transmitting antenna in periodically transmitting chirp signals with a frequency 100 Hz.

The probability of detection and probability of false alarm can be estimated by comparing the matched filter outputs to the corresponding threshold levels γ_m. The accuracy of this estimate

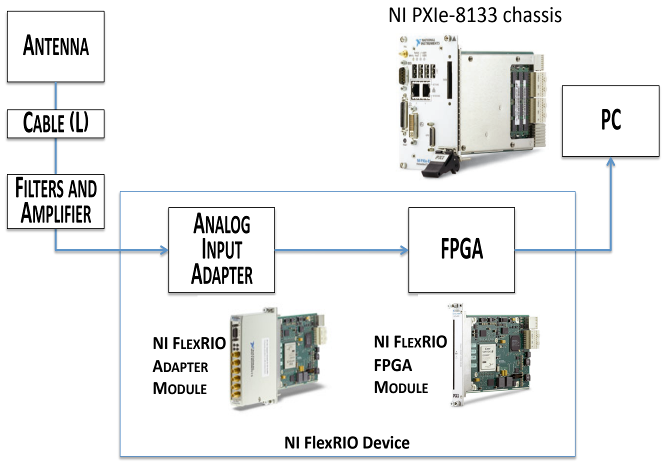

Fig. 5. Elements of the chirp detection receiver

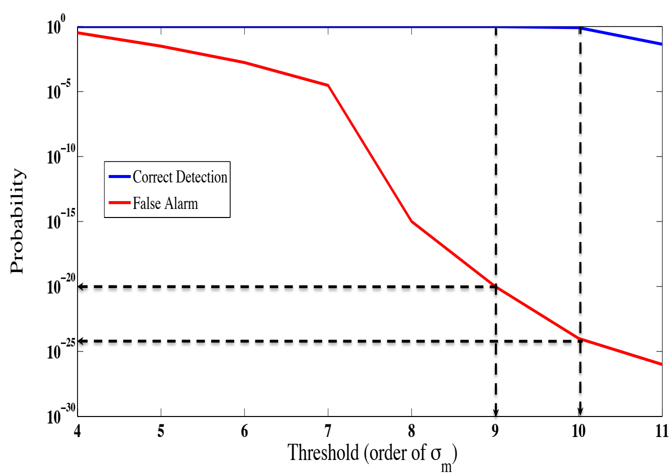

Fig. 6. Probability of Detection and False Alarm versus Threshold $(n * \sigma_m)$ under 0 dB SNR

will increase by increasing the number of considered samples or in other words the interval of time. Fig. 6 shows the calculated probability of false alarm and probability of detection over 10 minutes interval of time versus a range of threshold values under 0 dB received SNR. As depicted in the figure, we can find a range of threshold values $[9\sigma, 10\sigma]$ where probability of false alarm is in order of 10^{-20} or lower and at the same time can get complete probability of detection.

V. CONCLUSION

In the context of this paper, we addressed the primary phase of a promising bi-static radar approach in our challenging astrophysical problem. We presented a rake-like receiver that consists of a bank of matched filters for the detection of wide-band, short time chirp echoes (order of microseconds) which are produced by cosmic ray induced air showers. In the low SNR regime (-10 dB $\leq$ SNR $\leq$ 5 dB), our numerical analysis has shown that logarithmic assignment of chirp rates to the bank of matched filters results in near-optimal performance for chirp signals with uniformly distributed chirp rate. We briefly discussed system implementation using NI FlexRIO FPGA and demonstrated some experimental results. We have shown the system's ability to have complete probability of detection and probability of false alarm in order of 10^{-25} by choosing the threshold in the interval $[9\sigma_m$,$10\sigma_m]$ under 0 dB received SNR.

VI. REFERENCES

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